IJMRT: Volume (5), Issue 6, 2023
International Journal of Multidisciplinary Research Transactions
(A Peer Reviewed Journal)
www.ijmrt.in

ISSN (Print):2663-2381
ISSN(Online):2663-4007

Compressed Sensing Image Reconstruction Using Efficient Algorithm

Sathya. M[1*] , Gopperumdevi. G[2]

[1,2]Assistant Professor, Department of Electronics and Communication Engineering , Sri Bharathi Engineering College for Women, Pudukkottai, Tamilnadu, India.

[*]Corresponding author

DoI: https://doi.org/10.5281/zenodo.7883210

Abstract

Compressed sensing is a new information sampling theory and it's done for acquiring sparse (or) compressible data with much fewer measurements. This is particularly important for some imaging applications such as magnetic resonance image or in astronomy. In many practical situations, the noise behavior is impulsive and that the probability density function has very complex calculation than Gaussian. This motivates a number of impulsive noise suppression methods. Therefore, a new method is called robust CS is applied, following the principle of robust statistics which is using a convex but quadratic cost function on the residuals. By using a robust cost function on the residuals, we are able to suppress large outliers in the measurement noise. It also shows that an iterative algorithm can be developed under minimization– majorization framework and have established a theoretical guarantee on the improvement of the upper bound of the recovery error. The proposed method can be used to improve Cs recovery depend upon an inspection of the residuals for impulsiveness.

Keywords: Image Compression, Robust statistics, Compressed sensing.

1. Introduction

1.1. Image Compression

This thesis develops algorithms and applications in an emerging topic of signal processing called compressive sensing. Compressive sensing developed from questions raised about the efficiency of the conventional signal processing pipeline for compression, coding and recovery of natural signals, including audio, still images and video. The usual sequence of steps involved includes the following. First, the analog signal is sampled by a sensor such as a camera to obtain a sufficiently large number of digital samples. Second, the digitized samples are transformed into a suitable domain to compact the energy (and hence the information) into a relatively small number of numbers, called coefficients. The transformation is chosen to approximate the optimal Karhunen-Loeve transform and results in a representation of the original signal as a linear sum of a set of bases weighed by the coefficients. Most of the coefficients are small in magnitude and only a few coefficients contain a significant amount of energy. This implies that most of the information in the signal is concentrated in only a few bases of the signal. Third, this sparsity of transform coefficients is exploited to efficiently code the locations of the few large coefficients, and the magnitudes of these large coefficients are quantized and entropy coded. Finally, the coded representation is stored and/or transmitted to a decoder, where the coding and transformation steps are reversed to obtain a good approximation of the original set of digital samples, which can be used for D/A conversion and presentation to a viewer, with a quality close to that of the original sampled scene.

Figure.1. Left: Original image size 512x512; Right: Reconstructed image using10000 largest magnitude coefficients

This model is followed by all modern lossy compression algorithms for audio, still images and video, including the JPEG and JPEG2000 standards for still images [1,2], the Set Partitioning in Hierarchical Trees (SPIHT) algorithm for still image coding [3] , and the MPEG and H.264 standards [4] , [5] for video compression. The JPEG standard and MPEG standards use the 8x8 pixel Discrete Cosine Transform (DCT) to obtain energy compaction and de correlation, while the JPEG2000 standard and the SPIHT coder use a wavelet basis. Even if only a relatively small number of the largest magnitude coefficients is transmitted to the decoder, and the remaining are assumed to be zero, a good reproduction of the original image is obtained when the transform is inverted. Hence it is sufficient to transmit information only about the most significant coefficients to the receiver. This raises the following question: If only a few of the transform domain coefficients are needed for an acceptable reproduction, is it possible to bypass the step of recording a large number of samples, transforming them, and then throwing away all the insignificant coefficients Can one instead obtain the significant coefficients directly.

1.1.1. Overview of Comprehensive Sensing

Consider an underdetermined system $y=\Phi$ where Φ with , is a $M<N$ N-dimensional signal and y is a length vector of measurements equal to linear combinations of c . Suppose

that has nonzero elements, and we wish to recover from , One possible technique is to consider every subset Φ of columns drawn from Φ and test whether it fits by least squares leaving no residue. However this requires testing of C(N,S) subsets, which is infeasible for even moderate values of and S. show that if has nonzero elements with S=M/2and thematrix Φ satisfies some additional conditions, then can be recovered either exactly or with a small approximation error. For example, it is shown in [7] that if matrix Φ satisfies a Restricted Isometry Property (RIP), then minimization can recover the vector.

$$(1-\delta)\,\|c\|_2^2 \leq \|\Phi_I c\|_2^2 \leq (1+\delta)\,\|c\|_2^2 \qquad (1.1)$$

for every size m subset I of columns of Φ . If Φ satisfies the RIP with M=2S andthen can be recovered perfectly by solving

$$\min \ \|c\|_1 \ \text{ such that } y = \Phi c \qquad (1.2)$$

If is not exactly sparse, but the components decay rapidly in magnitude, then C can be approximately recovered with a distortion that is bounded by

$$\|c - c^*\|_2 \leq \frac{C_0}{\sqrt{S}} \|c - c_S\|_1 \qquad (1.3)$$

Where C0 is a small constant. The linear program in Equation (1.2) is a convex optimization problem that can be solved efficiently by interior point methods. However it is difficult to prove that a matrix Φ satisfies the RIP, and for large signals the convex optimization can still be computationally slow.

1.2. The Incoherence Parameter

An alternative formulation to Restricted Isometry has been defined in [8] that lower bounds the number of samples needed for perfect recovery using an incoherence parameter μ. Suppose that V (size n x n) is an orthogonal matrix satisfying VTV=nI and *V* . Select any *M* rows from V, to give the M x N matrix Φ as before. If the signal *c* has *m* nonzero values that are ±1, and if 20 *M* and also for some constants C0 and C1, then with probability exceeding 1-δ, thesignal *c* can be recovered by solving the same ***l***1- minimization mentioned above.

1.3. Example Of Recovery Using L1 Minimization

We illustrate the results above with some examples. The RIP property is satisfied with high probability for Gaussian matrices, i.e., matrices with entries drawn from a Gaussian distribution . We construct a size 128 x 200 matrix U with entries drawn from a 0-mean Gaussian distribution with variance 1/128. This makes *i E U* for all i, where Ui denotes the ith column of U and form a sparse vector c with 40 nonzero entries drawn from a random distribution. This is used to get , a length 128 sized sample vector. Then use l1-minimizationas described above to recover the signal x. We show the original signal and the recovered signal in Fig. 1.2.

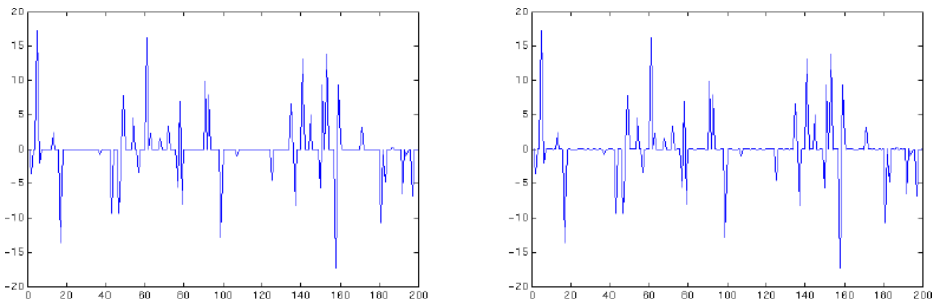


Figure. 1.3. Original signal with 40 nonzero entries on left, recovered signal on the right

A second approach to this problem involves greedy algorithms such as Orthogonal Matching Pursuit (OMP) [9] and its variants [10] [11] [12] [13]. In these algorithms, the projection Φ of the data is used to identify a single or a few bases that is/are believed to be in the true signal, and then the component of the data that is spanned by all the bases selected so far is removed, leaving behind a residue that is orthogonal to the bases selected. The residue is then used to identify more bases using Z Tr. The Orthogonal Matching Pursuit algorithm is listed in Fig. 1.3.

Algorithm: Orthogonal Matching Pursuit(Φ,y,S)
// Input: M by N matrix Φ, sample vector $y = \Phi c$, sparsity S
// Output: An estimate $\hat{c}$ for c
$r \leftarrow y, J \leftarrow \{\}, R \leftarrow \{1..N\}, t \leftarrow 0$
while $t < S$ **do**
    $z \leftarrow \Phi_R^T r$
    $d \leftarrow \text{argmax}_{j=1,\ldots,|R|} |z(j)|$
    $J \leftarrow J \cup \{R(d)\}, R \leftarrow R - \{R(d)\}$
    $\hat{c}_J \leftarrow \text{argmin}_x ||y - \Phi_J x||_2$
    $r \leftarrow y - \Phi_J \hat{c}_J$
    $t \leftarrow t + 1$
end while

Figure.1.3. Orthogonal Matching Pursuit

Compressed sensing (CS) has emerged as a new framework for signal acquisition and sensor design. CS enables a potentially large reduction in the sampling and computation costs for sensing signals that have a sparse or compressible representation. While the Nyquist-Shannon sampling theorem states that a certain minimum number of samples is required in order to perfectly capture an arbitrary band limited signal, when the signal is sparse in a known basis we can vastly reduce the number of measurements that need to be stored.

In recent years, compressed sensing (CS) has attracted considerable attention in areas of applied mathematics, computer science, and electrical engineering by suggesting that it may be possible to surpass the traditional limits of sampling theory. CS builds upon the fundamental fact that we can represent many signals using only a few non-zero coefficients in a suitable basis . Nonlinear optimization can then enable recovery of such signals from very few measurements.

Unfortunately, in many important and emerging applications, the resulting Nyquist rate is so high that we end up with far too many samples. Alternatively, it may simply be too costly, or even physically impossible,to build devices capable of acquiring samples at the necessary rate. Thus, despite extraordinary advances in computational power, the acquisition and processing of signals in application areas such as imaging, video, medical imaging, remote surveillance, spectroscopy, and genomic data analysis continues to pose a tremendous challenge.

The logistical and computational challenges involved in dealing with such high-dimensional data, we often depend on compression, which aims at finding the most concise representation of a signal that is able to achieve a target level of acceptable distortion. One of the most popular techniques for signal compression is known as transform coding, and typically relies on finding a basis or frame that provides sparse or compressible representations for signals in a class of interest .

By a sparse representation, we mean that for a signal of length n, we can represent it with k << n nonzero coefficients; by a compressible representation, we mean that the signal is well-approximated by a signal with only k nonzero coefficients.Both sparse and compressible signals can be represented with high fidelity by preserving only the values and locations of the largest coefficients of the signal. This process is called sparse approximation, and forms the foundation of transform coding schemes that exploit signal sparsity and compressibility, including the JPEG, JPEG2000, MPEG, and MP3 standards.

2. Mathematical Modeling

2.1. Optimal Condition

The first-order optimality condition of is ,

$$0 \in \partial E(x^*),$$

where $\partial E(x*)$ is the subdifferential of $E(\cdot)$ at $x*$ .We can apply the general property

$$y \in \partial f(x) \Leftrightarrow x \in \partial f^*(y),$$

for a convex function *f* and its convex conjugate

$$f^*(y) := \sup_x \{\langle y, x\rangle - f(x)\}$$

and get the equivalent condition: $x*$ is optimal if and only if there exists an auxiliary variable $y* = (y* ij)$

where $y_{ij} \in R2$, such that

$$0 \in \alpha\Phi \sum_{ij} L_{ij}^* y_{ij}^* + \beta\partial g(x^*) + \nabla_x h(x^*)$$
$$L_{ij}\Psi x^* \in \partial f^*(y_{ij}^*),$$

assume that signal $X \in R^N$ is s-sparse in basis Ψ=**I.** by using **RIP** constant ,The sparese vector $X \in R^N$ means,

$$(1 - \delta_S)||\mathbf{x}||_2^2 \leq ||\mathbf{\Phi x}||_2^2 \leq (1 + \delta_S)||\mathbf{x}||_2^2. \quad \ldots\ldots\ldots\ldots (2)$$

error upper bounded by $O(\|n\|2)$

$$(\mathcal{P}_1)\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^N} ||\mathbf{y} - \mathbf{\Phi x}||_2^2 + \lambda||\mathbf{x}||_1 \quad \ldots\ldots\ldots\ldots (3)$$

for a suitable value of the regularization parameter λ. by adjusting the λ the effect of outliers can be reduced.

The equation (3) is based on two goals,

- Minimizing the l2_ norm of the residuals
- Minimizing the l1 norm of the signal.

When the noise is Gaussian, this objective function is optimal in the maximum likelihood sense. However,when the noise is impulsive, the theory of robust statistics

Indicates that it would be prone to larger errors. The theory of robust statistics1 that a betterstrategy is to replace the quadratic cost function on residual $\|Y-\varphi x\|_2^2$

$$0 \in \alpha\Phi\sum_{ij} L_{ij}^{*} y_{ij}^{*} + \beta\partial g(x^{*}) + \nabla_x h(x^{*}), \quad (9)$$

$$L_{ij}\Psi x^{*} \in \partial f^{*}(y_{ij}^{*}), \quad (10)$$

$$f(\mathbf{x}) = g(\mathbf{x}) + \lambda\|\mathbf{x}\|_1$$

and the robust CS recovery is obtained by solving

$$(\mathcal{P}_1^r)\ \hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^N} f(\mathbf{x}).$$

2.2 Block Diagram Of The Project

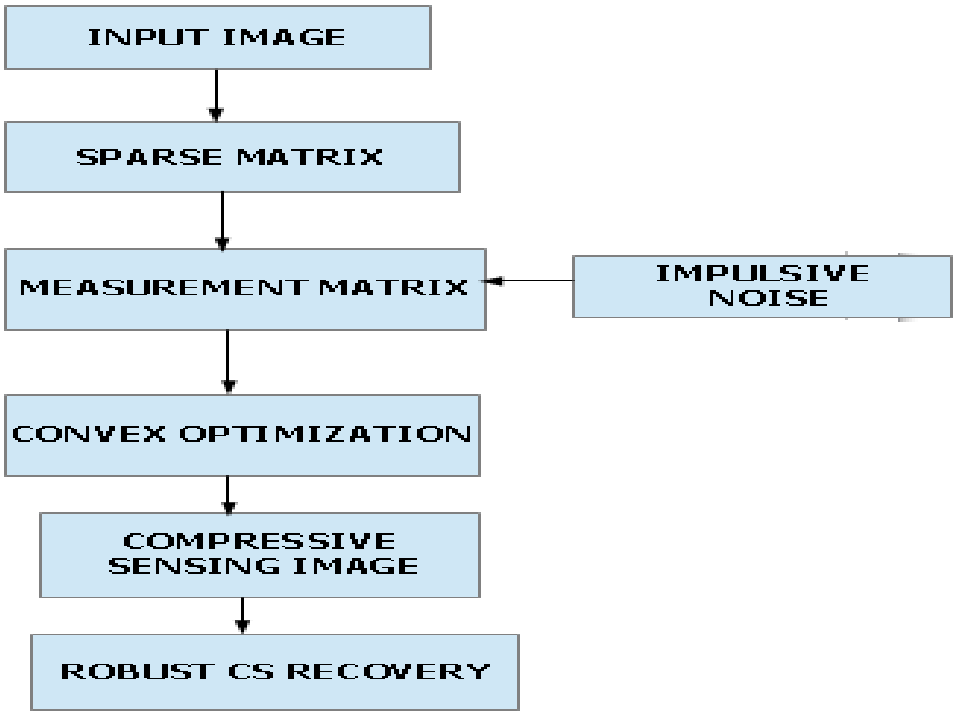


Figure.2.2. Block diagram of the project

The purpose of this project is to examine the quality of the recovered images from CS samples under the influence of impulsive noise. To get the Compressive sensing image the Sparse and Measurement matrix would be Computed then the proposed Robust CS recovery method would be computed.

2.3. Pre-Processing

The input Image would be the MRI image contaminated with the impulse noise and then the input image shall be scaled with respect to the coefficients mask .the Sparsity level would be the input for Measurement matrix.

2.4. Inputs

The MATLAB code shall have the following as user inputs, Input Image

2.4. Outputs

The MATLAB code shall have the following as user outputs,

- Compressive sensing Image
- Robust CS recovery image

2.5. Algorithms Involved

- Minimization Algorithm
- Convex Optimization
- Robust CS Recovery

2.6. Modules Of The Block Diagram

The base paper has been segregated into following modules,

Module I:

- CS Recovery methodModule II:
- Robust CS Recovery

2.7.CS Recovery

The main goal of this project is to carry out an empirical analysis of the rate-distortionperformanceof CS in image compression. If the impulsive noise enters a CS imaging application at the sensory level, the compressed data will be contaminated. The issues such as the minimization algorithm used and the transform employed, as well as the trade-off between number of measurements and quantization error. The measurement matrix shall be computed by using Fourier ,Bernoulli and uniform etc..By using the measurement matrix and Sparse matrix the CS formulation would be computed. The L2 minimization method one of the minimization method which is used for Standard CS recovery method. The Residual error would be large in L2 norm minimization method due to outliers. The Recovery efficiency of an L2 norm minimization also is low because of the residual error.

2.8. ROBUST CS RECOVERY

As mentioned above the recovery efficiency of the standard compressive sensing would be small so the output image computed by using standard CS formulation would be the low PSNR image. If the PSNR value is low means the image clarity would be low. So to get the high PSNR therobust compressive sensing algorithm would be computed .This problem involves in integrating robust statistics and the CS theory. So the proposed method for CS called robust CS, following the principle of robust statistics which is using a convex but sub quadratic cost function on the residuals. Finally the PSNR of the recovered image by the proposed CS formulation has a slightly higher value than that obtained by thestandard CS formulation. The Robust CS Recovery method can reduce effectively the recovery error bound and show that the improvement is related directly to the portion and the strength of the outliers in the noise samples.

3. Fast Composite Splitting Algorithms

FCSA decomposes the difficult composite regularization problem into multiple simpler sub problems and solve them in parallel. Each sub problems can be solved by the FISTA, which requires only O(1/√€) iterations to obtain an € optimal solution.

Algorithm 1. FCSA

Input: q = 1/L, t1 = 1 r1 = x0

repeat

for k = 1 to K do

for i = 1 to m do

x_i^k = proxp(g)(B (r^k ... - 1/L ∆f (r^k ...

end for

$x^k = 1/m \sum^m (B_i^{-1} y_i^k)$

$t^{k+1} = 1+\sqrt{1+4(t^k)^2}/2$

$r^{k+1} = x^k + t^{k+1}\ t^{k-1}/t\ (x^k - x^{k-1})\ x^k$

end for

until Stop criterions

In this algorithm, if we remove the acceleration step by setting $t^{k+1} \equiv 1$ in each iteration, we obtain the CSA. A key feature of the FCSA is its fast convergence performance borrowed the FISTA. we know that the FISTA can obtain an optimal € solution in O(1/√€).Another key feature of the FCSA is that the cost of each iteration may be O(mp log(p) under the following conditions: (1) the step y^k = proxp(g)(B (r^k i i - 1/L ∆f(r^k))) can be computed with the costO(p log(p)) for some prior models gi if Bi can be computed with O(p log(p)); (2) the step $x^k = 1/m \sum^m (Bi^{-1}\ y^k$ can also be computed with the cost of O(p log(p)) in these cases; and (3) other steps only involve adding vectors or scalars, thus cost only O(p) or O(1).

3.1. Enhancement

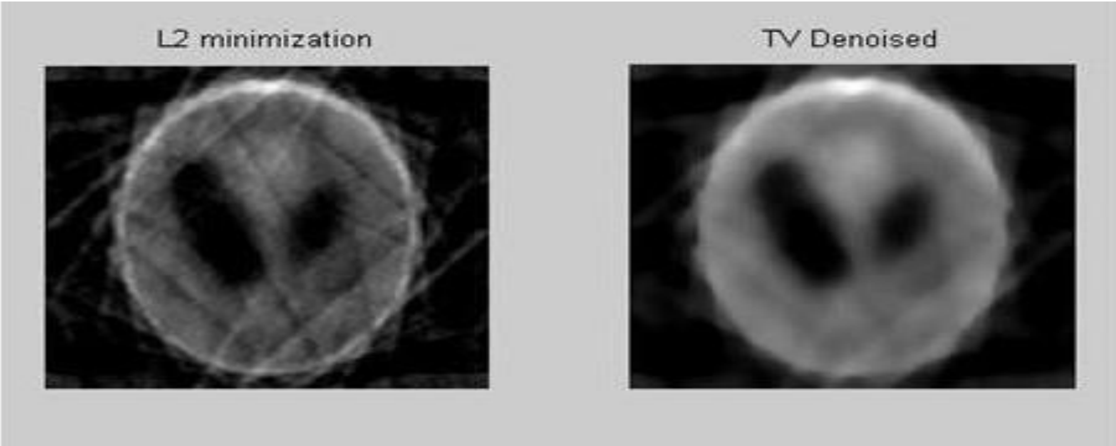
Here in this work the enhancement proposed gives a better result in terms of both accuracy and computational complexity. FCSA is more efficient than the TVCMRI and RecPF.It helps in getting a perfectimage without any noise and thus that image is given for futher process of image compression.

4. Result and Conclusion

I have presented a new approach to improving CS recovery in the presence of impulsive noise. By using a robust cost function on the residuals, we are able to suppress large outliers in the measurement noise. This results in an improved recovery because the regularization parameter is not influenced significantly by these outliers, meaning that the recovered signal is not being driven further toward 0. We also show that an iterative algorithm can be developed readily under the MM framework to utilize the powerand computational efficiency to obtain the solution of the new formulation.

Most importantly, i have established a theoretical guarantee on the improvement of the upper bound of the recovery error. The numerical studies on both synthetic and real images show that the proposed formulation achieves equivalent performance when the noise is indeed Gaussian, but an improvement is found when the noise is heavily impulsive.The proposed method can be used to improve further CS recovery upon an inspection of the residuals for impulsiveness.

4.1 Minimization Output Image



(a) Output Of L2 Minimization (b) TV Denoised

4.2 SIMULATION OUTPUT OF CSA AND FCSA

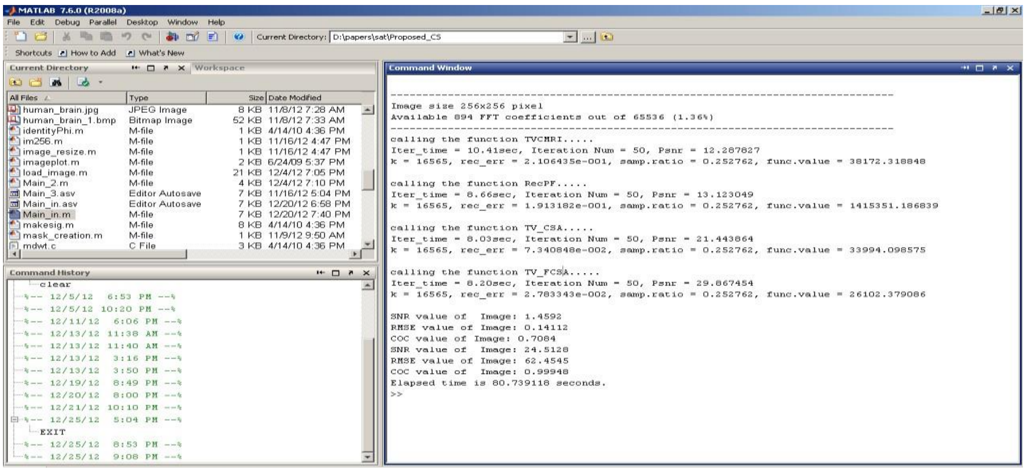


Table.1. Comparison Performance of *L*2 minimization and FCSA Methods

Image Parameters	SNR Values	PSNR Values	RMSE Values	COC Values
L2 Minimization	1.4592	12.2878	0.1411	0.7084
Fast Composite Splitting Algorithm	24.5128	29.8675	62.4545	0.9995

REFERENCES

[1]. Kissan G. Gauns Dessai; Venkatesh V. Kamat; Ramrao S. Wagh et al Proposed" Effective Use of Rubrics in Computer Assisted Subjective Answer-Script Evaluation" IEEE-2020.

[2]. A. Auslender and M. Teboulle, "Interior gradient and proximal methods for convex and conic optimization," SIAM J. Optim., vol. 16 no. 3, pp. 697–725, 2006.

[3]. L. Bar, A. Brook, N. Sochen, and N. Kiryati, "Deblurring of color images corrupted by impulsive noise," IEEE Trans. Image Process., vol. 16, no. 4, pp. 1101–1111, Apr. 2007.

[4]. A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," SIAM J. Imag. Sci., vol. 2, no. 1,pp. 183–202, Jan. 2009.

[5]. J. Bobin, J.-L. Starck, and R. Ottensamer, "Compressed sensing in astronomy," IEEE J. Sel. Topics Signal Process., vol. 2, no. 5, pp.718–726, Oct. 2008.

[6]. S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, U.K.: Cambridge Univ. Press, 2004.

